

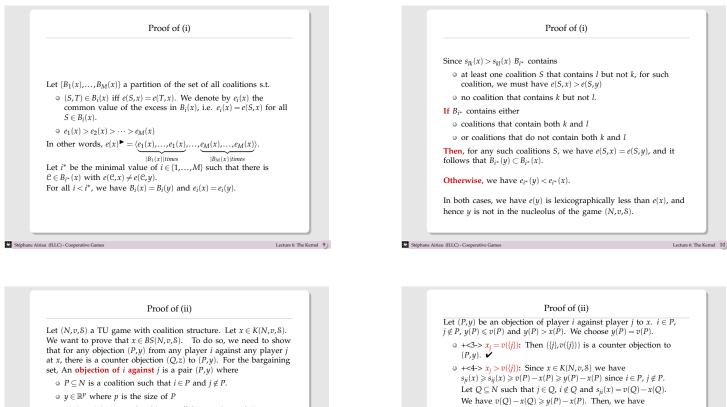
Properties	Proof of (i)
	Let $x \notin K(N, v, S)$, we want to show that $x \notin Nu(N, v, S)$.
eorem	$f(X(X = 0))$ have then with $0 \in CC$ and $(h, l) \in Q^2$ such that
Let (N, v, S) a game with coalition structure, and let	$x \notin K(N, v, \delta)$, hence, there exists $\mathcal{C} \in CS$ and $(k, l) \in \mathcal{C}^2$ such that $s_{lk}(x) > s_{kl}(x)$ and $x_k > v(\{k\})$.
$\mathfrak{I}mp \neq \emptyset$. Then we have:	Let y be a payoff distribution corresponding to a transfer of utility
• (i) $Nu(N,v,S) \subseteq K(N,v,S)$	$\begin{cases} x_i \text{ if } i \neq k \text{ and } i \neq l \end{cases}$
• (ii) $K(N,v,S) \subseteq BS(N,v,S)$	$\epsilon > 0 \text{ from } k \text{ to } l: \ y_i = \begin{cases} x_i \text{ if } i \neq k \text{ and } i \neq l \\ x_k - \epsilon \text{ if } i = k \\ x_l + \epsilon \text{ if } i = l \end{cases}$
	Since $x_k > v(\{k\})$ and $s_{lk}(x) > s_{kl}(x)$, we can choose $\epsilon > 0$ small
eorem	 enough s.t.
Let (N, v, S) a game with coalition structure, and let	• $x_k - \epsilon > v(\{k\})$
$\exists mp \neq \emptyset$. The kernel $K(N,v,\mathcal{S})$ and the bargaining set $BS(N,v,\mathcal{S})$ of the game are non-empty.	$\circ s_{lk}(y) > s_{kl}(y)$
of	 We need to show that $e(y)^{\blacktriangleright} \leq_{lex} e(x)^{\blacktriangleright}$.
Since the Nucleolus is non-empty when $\Im mp \neq \emptyset$, the	
proof is immediate using the theorem above.	Note that for any coalition $S \subseteq N$ s.t. $e(S, x) \neq e(S, y)$ we have either
	• $k \in S$ and $l \notin S$ $(e(S,x) > e(S,y)$ since $e(S,y) = e(S,x) + \varepsilon > e(S,x)$
	• $k \notin S$ and $l \in S$ $(e(S,x) < e(S,y)$ since $e(S,y) = e(S,x) - \epsilon < e(S,x)$

el 8)

Theorem

Theorem

Proof



- $y(P) \leq v(P)$ (y is a feasible payoff for members of P)
- $\forall k \in P, y_k \ge x_k \text{ and } y_i > x_i$
- An **counter-objection to** (P,y) is a pair (Q,z) where
- $Q \subseteq N$ is a coalition such that $j \in Q$ and $i \notin Q$.
- $z \in \mathbb{R}^q$ where *q* is the size of *Q*
- $z(Q) \leq v(Q)$ (z is a feasible payoff for members of Q)
- $\forall k \in O, z_k \ge x_k$
- $\forall k \in Q \cap P \ z_k \ge y_k$

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Computing a kernel-stable payoff distribution • There is a transfer scheme converging to an element in the kernel. It may require an infinite number of small steps. We can consider the ε-kernel where the inequality are
defined up to an arbitrary small constant ε . R. E. Stearns. Convergent transfer schemes for n-person games. Transactions of the American Mathematical Society, 1968. Stéphane Airiau (ILLC) - Cooperative Gam Lecture 6: The Kernel 13

Lecture 6: The Kernel 11)

- The complexity for one side-payment is $O(n \cdot 2^n)$.
- Upper bound for the number of iterations for converging to an element of the ϵ -kernel: $n \cdot log_2(\frac{\delta_0}{\epsilon \cdot v(S)})$, where δ_0 is the maximum surplus difference in the initial payoff distribution.
- To derive a polynomial algorithm, the number of coalitions must be bounded. For example, only consider coalitions which size is bounded in $[K_1, K_2]$. The complexity of the truncated algorithm is $O(n^2 \cdot n_{coalitions})$ where $n_{coalitions}$ is the number of coalitions with size in $[K_1, K_2]$, which is a polynomial of order K_2 .
- M. Klusch and O. Shehory. A polynomial kernel-oriented coalition algorithm for rational information agents. In Proceedings of the Second International Conference on Multi-Agent Systems, 1996. • O. Shehory and S. Kraus. Feasible formation of coalitions among autonomous agents in non-superadditve environments. Computational Intelligence, 1999.

Computing a kernel-stable payoff distribution Algorithm 1: Transfer scheme converging to a ϵ -Kernelstable payoff distribution for the CS § compute-c-Kernel-Stable(N, v, S, c) **for** each member $(i,j) \in \mathbb{C}, i \neq j$ **do** // compute the maximum surplus $s_{ij} \leftarrow \max_{R \subseteq N \mid (i \in R, j \notin R)} v(R) - x(R)$ $\delta \leftarrow \max_{(i,j) \in C^2, C \in S} s_{ij} - s_{ji}$ $\begin{array}{l} (i^{\star},j^{\star}) \leftarrow \operatorname{argmax}_{(i,j) \in \mathbb{N}^2} (s_{ij} - s_{ji});\\ \text{if } (x_{j^{\star}} - v(\{j\}) < \frac{\delta}{2}) \text{ then }\\ \downarrow \quad d \leftarrow x_{j^{\star}} - v(\{j^{\star}\}); \end{array}$ // payment should be individually rational else $d \leftarrow \frac{\delta}{2};$ $\begin{array}{c} x_{i^{\star}} \leftarrow x_{i^{\star}} + d; \\ x_{i^{\star}} \leftarrow x_{i^{\star}} - d; \end{array}$ until $\frac{\delta}{v(S)} \leqslant \epsilon$; Lecture 6: The Kernel 14

 $v(Q) \ge y(P) + x(Q) - x(P)$

Finally $x \in BS(N, v, S)$.

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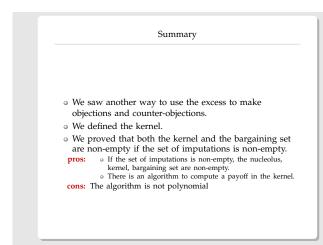
 $\geq \quad y(P\cap Q)+y(P\setminus Q)+x(Q\setminus P)-x(P\setminus Q)$

Let us define z as follows $\begin{cases} x_k \text{ if } k \in Q \setminus P \\ y_k \text{ if } k \in Q \cap P \end{cases}$

(Q,z) is a counter-objection to (P,y).

 $y(P \cap Q) + x(Q \setminus P)$ since $i \in P \setminus Q$, $y(P \setminus Q) > x(P \setminus Q)$

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 The Shapley value. It is not a stability concept, but it tries to guarantee fairness. We will see it can be defined axiomatically or using the concept of marginal contributions.

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